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Model-Based Feedforward Precompensation and VS-Type Robust Nonlinear Postcompensation for Uncertain Robotic Systems with/without Knowledge of Uncertainty Bounds(I)

Sam-Sang You* and Seok-Kwon Jeong**

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In this paper, a robust motion tracking control algorithm for robotic manipulators is proposed where the higher-order system uncertainties are taken into account. The control structure consists of two main parts: a model-based precompensation part and a robust nonlinear controller one. Specifically, with knowledge of possible upper bounds on uncertainties, we propose the nonadaptive version of robust controller. Stability and robustness issues of the controllers have been investigated via a Lyapunov method and it is shown that the proposed control algorithms are highly robust in the presence of significant system uncertainties. Finally, the computer simulation results are presented to validate the proposed algorithm.

Key Words: Uncertain Robot, Motion Tracking, Precompensation (Postcompensation), Variable Structure (VS), Robust Nonlinear Control, Deterministic Approach, Norms, Structured (Unstructured) Uncertainties, Lyapunov Stability, Uniform Ultimate Boundedness (UUB), Chattering.

Nomenclature

Symbols written in bold type denote vectors or matrices, while scalars are written normally:

R^+ : Set of non-negative real number; $R^+ := [0, +\infty)$

R^n : n-dimensional vector space with real elements R

$R^{n\times m}$: Set of all real-valued $(n\times m)$ matrices

inf : Infimum, the greatest lower bound

sup : Supremum, the least upper bound

$\mathbf{x}$: A vector; $\mathbf{x}=[x_1\ x_2 \cdots x_n]^T$, $x_i \in R$

$\|\mathbf{x}\|$: Euclidean norm of a vector $\mathbf{x}$; $\|\mathbf{x}\| = [\mathbf{x}^T\mathbf{x}]^{1/2}$, $\forall \mathbf{x}\in R^n$

$\mathbf{A}>0(<0)$: Positive (negative) definite matrix $\mathbf{A}$

$\|\mathbf{A}\|$: Induced norm of a real matrix $\mathbf{A}\in R^{n\times m}$; $\|\mathbf{A}\| = [\lambda_{\max}(\mathbf{A}^T\mathbf{A})]^{1/2}$

$\Omega_r(x)$: Closed ball in R^n of radius $r\in R^+$ centered at $\mathbf{x}=0$: $\Omega_r(\mathbf{x}) = (\mathbf{x}\in R^n : \|\mathbf{x}\| \le r)$

C^p : Set of p-times continuously differentiable functions

$\mathbf{E}_n$: $(n\times n)$ Identity matrix

L_p : The function norm in the Lebesgue space; Let $\mathbf{f}(t): R^+ \rightarrow R^n$ be Lebesgue measurable function, then the L_p-norm $\|\mathbf{f}\|_p$ is defined as $\|\mathbf{f}\|_p = [\int_0^\infty \|\mathbf{f}(t)\|^p dt]^{1/p} < \infty$, for $p\in[1, \infty)$. When $p=\infty$, $\mathbf{f}\in L_\infty$ if and only if $\|\mathbf{f}\|_\infty = \sup_{t\in[0,+\infty)} \|\mathbf{f}(t)\| < \infty$

$\lambda_{\max}(\mathbf{A})$: Maximum eigenvalue of $\mathbf{A}$; $\lambda_{\max}(\mathbf{A}) = \max_i\{\lambda_i(\mathbf{A})\}$, where $\lambda_i(\mathbf{A})$ is the ith eigenvalue of matrix $\mathbf{A}$

$\lambda_{\min}(\mathbf{A})$: Minimum eigenvalue of $\mathbf{A}$; $\lambda_{\min}(\mathbf{A}) = \min_i\{\lambda_i(\mathbf{A})\}$

$(\bullet)^C$: Complement of $(\bullet)$

* Professor, Department of Mechanical Engineering, Korea Maritime University, Yeongdo-Ku, Pusan 606-791, Korea

** Research Center for Ocean Industrial Development, Pukyong National University 599-1 Daeyeon-dong, Nam-gu, Pusan, Korea.

1. Introduction

Currently, robotic manipulators are often expected to solve a wide variety of automation needs in many modern industries. Thus, the development of effective (and reliable) control algorithms will be an important step toward high-speed and -precision robot system.

Research has shown that many physical systems contain various uncertainties, which include structured and unstructured uncertainties. Several model-based control schemes, such as inverse dynamics and passivity-based controllers, have been presented for motion control of robots. Craig et al. were one of the first to examine this approach in detail. Among the various approaches to improving system performance under parametric uncertainties, adaptive control method (Craig, 1987 ; Slotine, 1987 ; Ortega, 1989 ; Sadegh, 1990 ; Chen, 1990) has been used extensively. During last decade, several research papers dealing with the control of uncertain dynamical systems have been published. In this paper, based on the deterministic approach, the robust control method for uncertain systems is discussed for real-time applications. One of the useful design method is variable structure (VS)-type control schemes.(Qu, 1992 ; Spong, 1992 ; Abdallah, 1991 ; You, 1994a ; You, 1994b) Abdallah et al. surveyed the robust control of robotic manipulators. Although some works have been done in this area to date, more studies still need to be conducted to ascertain the effectiveness of a control system under higher-order uncertainties. Specifically, this paper suggests a decentralized control method. Up to this point, many advanced control strategies employed fall into the following categories: cope with relatively small uncertainties; utilize computationally complex algorithms; synthesize purely discontinuous controllers.

The aim of this investigation is to study the robust trajectory tracking controllers for an uncertain robotic system, which will overcome all the defects in earlier methods. The control algorithm consists of two major components: the nominal (or primary) control and the robust nonlinear controller. The uncertainties assumed are bounded by higher-order polynomials in the norms of system states. The nominal values of robot parameters are used in the primary control law instead of updating model-parameters on-line (as in the centralized adaptive control approach). It is shown that all possible responses of the corresponding closed-loop system are at least uniformly ultimately bounded under significant uncertainties.

This paper is organized as follows. In Sec. 2, system description and formulation are presented. In Sec. 3, the robust nonlinear controller has been proposed with known uncertainty bounds. In Sec. 4, numerical simulations are conducted to test the performance of the closed-loop system, while the conclusions of the research are summarized in Sec. 5.

2. System Description and Formulation

Based on the Euler-Lagrangian dynamics, the mathematical model of n-DOF (degrees of freedom) rigid manipulator with open-loop chain can be written in compact vector-matrix form as follows : (Craig, 1987 ; Slotine, 1987 ; Ortega, 1989 ; Sadegh, 1990 ; Spong, 1992 ; Abdallah, 1991 ; You, 1994a ; You, 1994b)

$$\begin{aligned} &\boldsymbol{M}(\boldsymbol{q};\Theta)\ddot{\boldsymbol{q}}+\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}};\Theta)\dot{\boldsymbol{q}}+\boldsymbol{G}(\boldsymbol{q};\Theta)\\ &+\boldsymbol{T}_u(\boldsymbol{q},\dot{\boldsymbol{q}})=\boldsymbol{T},\ \forall t\geq 0 \end{aligned} \tag{1}$$

where $\boldsymbol{q}(\in R^n)$, $\dot{\boldsymbol{q}}$ and $\ddot{\boldsymbol{q}}$ are the vectors of generalized positions, velocities, and accelerations, respectively ; $\boldsymbol{M}(\boldsymbol{q};\Theta)\in R^{n\times n}$ is the inertia matrix ; $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}};\Theta)\in R^{n\times n}$ is the nonlinear coupling terms representing the centrifugal/Coriolis effect ; $\boldsymbol{G}(\boldsymbol{q};\Theta)\in R^n$ is the gravitational torque vector ; $\boldsymbol{T}\in R^n$ represents the generalized torque vector ; $\boldsymbol{T}_u(\boldsymbol{q},\dot{\boldsymbol{q}})\in R^n$ is the vector of the unstructured uncertainties whose functional structures are poorly known or completely unknown, such as friction, link and joint flexibilities, external disturbances, sensor and actuator noises, strengths of interactions from other subsystems, and other unmodelled dynamics ; $\Theta\in R^m$ represents the vector of bounded system parameters, such as link masses, link lengths, and moments of inertia.

It is well known that the dynamic model (1) having revolute joints satisfies the following essential properties.(Craig, 1987 ; Slotine, 1987 ; Ortega, 1989 ; Sadegh, 1990 ; Spong, 1992 ; Abdallah, 1991 ; You, 1994a ; You, 1994b ; You, 1994c)

[P1]: M is symmetric and positive-definite matrix, i.e., $M=M^T>0$, $\forall(q, \Theta)$. Furthermore, M(C^∞ function) is uniformly bounded as $\delta_l\leq\|M\|\leq\delta_u$, $\forall(q, \Theta)$, where δ_l and $\delta_u(<\infty)$ are some positive constants.

[P2]: A part of the dynamics (1) is linear in terms of a suitably selected set of system parameters

$$M(q;\Theta)\dot{x}+C(q,\dot{q};\Theta)x+G(q;\Theta)=Y(q,\dot{q},x,\dot{x})\Theta,\ \forall(q,\dot{q},x,\dot{x})\in R^n \quad (2)$$

where $Y\in R^{n\times m}$ is called the regressor matrix ; and the vector $\Theta\in R^m$ belongs to a bounded set.

[P3] : $x^T(\dot{M}-2C)x=0$, $\forall x\in R^n$. Namely, $(\dot{M}-2C)$ is a skew-symmetric matrix with

$$C(q, \dot{q};\Theta)=\begin{bmatrix}\dot{q}^T\overline{C}_1(q;\Theta)\\ \vdots \\ \dot{q}^T\overline{C}_i(q;\Theta)\\ \vdots \\ \dot{q}^T\overline{C}_n(q;\Theta)\end{bmatrix} \quad (3)$$

where $\overline{C}_i\in R^{n\times n}$ are symmetric and bounded matrices, which may be defined as

$$\overline{C}_i={}^1/_2\left[\frac{\partial m_i}{\partial q}+\left(\frac{\partial m_i}{\partial q}\right)^T-\frac{\partial M}{\partial q_i}\right]$$

where m_i denotes the ith column (or row) of M, and q_i is the ith component of q.

[P4]: $C(q, x)z=C(q, z)x$, $\forall(x, z, q)\in R^n$. Also, it is known that the norm of C satisfies $\|C\|\leq\alpha_1\|\dot{q}\|$, $\forall(q, \dot{q}, \Theta)$, with $\alpha_1\in R^+$.

[P5]: G(C^∞ function) is bounded by $\|G\|\leq\alpha_2$, where α_2 is a scalar constant.

Now, consider the following assumptions for the problem formulation.

[A1]: The vector $\Theta=[\theta_1\cdots\theta_i\cdots\theta_m]^T$ is not exactly known, but the variation of θ_i is within a prescribed range $\Psi_i=[\underline{\theta}_i, \overline{\theta}_i]\subset R$, where $\underline{\theta}_i$ and $\overline{\theta}_i$ are known positive constants. Therefore, we have $\Psi=\Psi_1\times\cdots\times\Psi_m$ in which Θ $(\in\Psi)$ is a nonempty and compact set.

[A2] : The desired trajectory ($q_d\in C^2$ function) and its derivatives are all continuous and uniformly bounded by

$$d_j=\sup_{t\in R^+}\left\|\frac{d^jq_d}{dt^j}\right\|,\ (j=0, 1, 2)$$

where d_j $(<\infty)$ are known positive constants.

A class of tracking errors are defined as follows : $e\in R^n$ is the vector of the position tracking errors, $e_p=q-q_d$, with $\dot{e}_v=de_p/dt$, where $q_d\in R^n$ is the desired position vector ; the reference tracking errors $\dot{e}_r\in R^n$ are defined by $\dot{e}_r=\dot{q}_d-Fe_p$, with $F=\varepsilon E_n$, where $\varepsilon(>0)$ is scalar constant; the sliding surface variable vector $e_s\in R^n$ is defined as $e_s=\dot{q}-\dot{e}_r$.

Lemma 1 : If $\|e_p(t)\|\leq\overline{d}$ $(<\infty$ is satisfied for any $t\in[t_0, \infty)$, then

$$\|e_p(t)\|\leq\exp[-\varepsilon(t-t_0)]\{\|e_p(t_0)\|-\overline{d}/\varepsilon\}+\overline{d}/\varepsilon \quad (4)$$

and

$$\|\dot{e}_v(t)\|\leq\overline{d}+\varepsilon\|e_p(t)\| \quad (5)$$

Proof: The proof is a straightforward.(You, 1994c)

In fact, this lemma shows that the norm bounds of e_p and $\dot{e}_v$ can be obtained from that of e_s, thus the corresponding ultimate bounds in (4) and (5) are respectively given by

$$\lim_{t\to\infty}\|e_p(t)\|=\overline{d}/\varepsilon \text{ and}$$
$$\lim_{t\to\infty}\|\dot{e}_v(t)\|=2\overline{d}$$

In the special case, if $\overline{d}=0$, then $\lim_{t\to\infty}\|e_p(t)\|\to 0$ and $\lim_{t\to\infty}\|\dot{e}_v(t)\|\to 0$

Thus, in this study the control objectives can be thought of as follows :

$$\lim_{t\to\infty} sup\|e_p(t)\|\leq\overline{r}_p \text{ and}$$
$$\lim_{t\to\infty} sup\|\dot{e}_v(t)\|\leq\overline{r}_v\forall t\geq 0$$

where the desired tolerances $\overline{r}_p$ and $\overline{r}_v(\in R^+)$ can be considered as a measure of closeness of the system tracking performance to the asymptotic stability $(\overline{r}_p=\overline{r}_v=0)$.(Sadegh, 1990; Chen, 1990; You, 1994c; Sastry, 1989) That is, the design objective is to formulate a control input

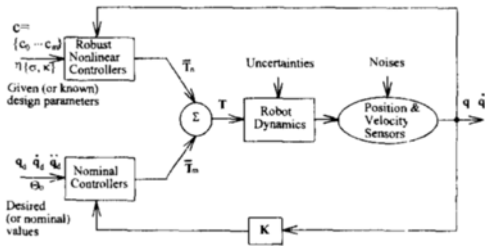

Fig. 1 Block diagram of the proposed control algorithm

vector so that the actual system responses track the desired quantities as closely as possible irrespective of significant uncertainties.

Now, the general control structure (see Fig. 1) is given in the following form (You, 1994a ; You, 1994b)

$$\boldsymbol{T}=\overline{\boldsymbol{T}}_m+\overline{\boldsymbol{T}}_n,\ t\geq 0 \tag{6}$$

which is the sum of two major parts. That is, a model-based feedforward precompensation (or a nominal controller) is chosen by

$$\overline{\boldsymbol{T}}_m=\boldsymbol{M}_0(\boldsymbol{q}_d\,;\,\Theta_0)\,\ddot{\boldsymbol{e}}_r+\boldsymbol{C}_0(\boldsymbol{q}_d,\ \dot{\boldsymbol{q}}_d\,;\,\Theta_0)\,\dot{\boldsymbol{e}}_r+\boldsymbol{G}_0(\boldsymbol{q}_d\,;\,\Theta_0)-\boldsymbol{K}\boldsymbol{e}_s \tag{7}$$

where $\boldsymbol{M}_0$, $\boldsymbol{C}_0$, $\boldsymbol{G}_0$, and Θ_0 denotes the nominal values of the true values $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{G}$, and Θ via modeling, respectively ; the feedback gain matrix $\boldsymbol{K}=KE_n$ is chosen by the system designer, with $(K>0)$. And an auxiliary control input (or a robust nonlinear control), $\overline{\boldsymbol{T}}_n\in R^n$, is intended to account for both the compensation error and the unstructured uncertainties, and its structure is specified later. In fact, the model-based portion can be calculated off-line, since the desired trajectories and the nominal values of dynamic parameters are known in advance. Substituting (6) into (1) along with some algebraic manipulations becomes

$$\boldsymbol{M}\dot{\boldsymbol{e}}_s=-\boldsymbol{C}\boldsymbol{e}_s-(\boldsymbol{T}_u+\boldsymbol{T}_s)-\boldsymbol{K}\boldsymbol{e}_s+\overline{\boldsymbol{T}}_n \tag{8}$$

where

$$\boldsymbol{T}_s=[\boldsymbol{M}-\boldsymbol{M}_0]\,\ddot{\boldsymbol{e}}_r+[\boldsymbol{C}-\boldsymbol{C}_0]\,\dot{\boldsymbol{e}}_r+[\boldsymbol{G}-\boldsymbol{G}_0] \tag{9}$$

which represents the structured uncertainties caused by system modeling errors (or parameter variations).

Remark : For the vector of nominal values $\Theta_0=[\theta_{01}\cdots\theta_{0i}\cdots\theta_{0m}]^T$, the i th nominal value θ_{0i} may be selected as $\theta_{0i}={}^1/_2(\underline{\theta}_i+\overline{\theta}_i)$, that is, the mean value of the admissible range of θ_i, otherwise by designer's convenience.

In deterministic control approach, the possible upper bounds on the system uncertainties can be expressed in the general form

$$\|\boldsymbol{T}_s\|\leq\bar{\omega}_s(t,\ \boldsymbol{e}_p,\ \dot{\boldsymbol{e}}_v)\ \text{and}$$
$$\|\boldsymbol{T}_u\|\leq\bar{\omega}_u(t,\ \boldsymbol{e}_p,\ \dot{\boldsymbol{e}}_v)$$

where $\bar{\omega}_s$ and $\bar{\omega}_u(\in R^+)$ are continuous and scalar bounding functions.

First, the following properties are stated for the system dynamics (1).

[P6] : There exist scalar bounding constants $\rho_{1i}\in R^+$, $\forall t\in R^+$, such that

$$\text{(i)}\ \rho_{11}=\sup_{(\Theta,\,\Theta_0)\in\Psi}\ \sup_{(q,q_d)\in R^n}\|\boldsymbol{M}-\boldsymbol{M}_0\|$$
$$\text{(ii)}\ \|\boldsymbol{C}-\boldsymbol{C}_0\|\leq\rho_{12}\|\dot{\boldsymbol{q}}\|+\rho_{13}\|\dot{\boldsymbol{q}}_d\|$$
$$\text{with}\ \rho_{12}=\sup_{\Theta\in\Psi}\ \sup_{q\in R^n}\textstyle\sum_{i=1}^n\|\overline{C}_i\|\ \text{and}$$
$$\rho_{13}=\sup_{\Theta_0\in\Psi}\ \sup_{q_d\in R^n}\textstyle\sum_{i=1}^n\|\overline{C}_{0i}\|$$
$$\text{(iii)}\ \rho_{14}=\sup_{(\Theta,\,\Theta_0)\in\Psi}\ \sup_{(q,q_d)\in R^n}\|\boldsymbol{G}-\boldsymbol{G}_0\|$$

If the augmented tracking error vector $\overline{\boldsymbol{z}}\in R^{2n}$ is defined as $\overline{\boldsymbol{z}}=[\boldsymbol{e}_p{}^T\dot{\boldsymbol{e}}_v{}^T]^T$, then the following assumption and lemmas provide the possible upper bounds on the uncertainties.

Lemma 2 : The structured uncertainties ($\boldsymbol{T}_s$) are bounded

$$\|T_s\|\leq a_0+a_1\|\overline{\boldsymbol{z}}\|+a_2\|\overline{\boldsymbol{z}}\|^2=\bar{\omega}_s$$

where $a_i(i=0,\,1,\,2)$ are known positive constants.

Proof : See Appendix A for the complete proof.

[A3] : The unstructured uncertainties ($\boldsymbol{T}_u$) are given in powers of $\|\overline{\boldsymbol{z}}\|$

$$\|\boldsymbol{T}_u\|\leq b_0+b_1\|\overline{\boldsymbol{z}}\|+\cdots+b_m\|\overline{\boldsymbol{z}}\|^m=\textstyle\sum_{i=0}^m b_i\|\overline{\boldsymbol{z}}\|^i=\bar{\omega}_u$$

where b_i $(i=1,\ \cdots,\ m)$ are known positive constants, and m is the highest order of $\bar{\omega}_u$ in the system uncertainties.

Lemma 3: The combined (or structured and unstructured) uncertainties are bounded by

$$\|\boldsymbol{T}_s+\boldsymbol{T}_u\|\leq\|\boldsymbol{T}_s\|+\|\boldsymbol{T}_u\|\leq c_0+c_1\|\overline{\boldsymbol{z}}\|+\cdots+c_m\|\overline{\boldsymbol{z}}\|^m=\textstyle\sum_{i=1}^n c_i\|\overline{\boldsymbol{z}}\|^i=\Phi_s$$

where $c_0(=a_0+b_0)$, $c_1(=a_1+b_1)$, $c_2(=a_2+b_2)$, and $c_i(=b_i\,;\ i=3,\ \cdots,\ m)$ are some known constants ; $\Phi_s(=\bar{\omega}_s+\bar{\omega}_u)\in R^+$ is continuous and known scalar bounding function.

In fact, the robust nonlinear control ($\bar{T}_n$) in the error dynamics (8) is intended to postcompensate for T_u and T_s (equivalently, Φ_s)

3. Synthesis of Robust Nonlinear Control

In this paper, the uncertainties under consideration are assumed to be known ; that is, *a priori* knowledge of uncertainty bound on Φ_s is required. If the information on Φ_s is unknown, then the adaptive mechanism, which will be discussed in the subsequent paper (Ⅱ), can be employed to estimate the system uncertainties.

The robust control structure with VS-type scheme (see Fig. 1) is given by (Qu, 1992 ; You, 1994a ; You, 1994b)

$$\bar{T}_n=-\frac{\Phi_s^2 e_s}{\| e_s \|\Phi_s+\eta},\ t\geq 0 \tag{10}$$

with $\eta(t)=\sigma\ \exp(-\kappa t)$, where the design parameters σ and κ are non-negative constants. Clearly, the existence of $\eta(t)$ in $\bar{T}_n$ guarantees the continuity of control input vector even when $\| e_s(t) \|$ becomes zero.

Remark : The VS-type nonlinear controller (10) can be discontinuous as $t\rightarrow\infty$, i.e., $\lim_{t\to\infty}\eta(t)\rightarrow 0$, but we are mainly interested in the trajectory tracking properties in a finite time.

The system stability and the tracking performance (including the robustness issues) are given in the following.

Theorem : With the known coefficients (c) on the uncertainty bounds in Lemma 3, the solutions to the closed-loop error dynamics (8) under (10) are globally stable (or bounded). That is, there exists the compact set (or closed and bounded set) Λ such that for all $(e_p(0),\ \dot{e}_v(0),\ e_s(0))\in\bar{\Lambda}$, every trajectory, with $\eta\neq 0(\sigma>0,\ \kappa=0)$, globally converges to the following residual set (or the ball of attraction) with the ultimate bound V_f :

$$\Lambda=\{e_s\in R^n:\ V(t,\ e_s)\leq V_f\},\ \forall t\in R^+$$

where $V_f=\sigma/\nu$ and $\nu=2K/\lambda_{\max}(M)$, with $\Lambda\subset\bar{\Lambda}$. Otherwise, all signals under the conditions that $\eta\neq 0(\sigma>0,\ \kappa>0)$ or $\eta=0(\sigma=0)$, are globally exponentially (or asymptotically) stable, i.e., $\lim_{t\to\infty}(e_p,\ \dot{e}_v,\ e_s)\rightarrow 0$.

Proof: The proof proceeds by choosing a Lyapunov-like function (at least a C^1 function), $v(t,\ e_s)$: $R^+\times R^n\rightarrow R^+$, which is given by

$$V={}^1/_2 e_s^T M e_s,\ M>0 \tag{11}$$

By introducing Rayleigh's principle,(You, 1994c) we observe that

$${}^1/_2\lambda_{\min}(M)\| e_s \|^2\leq V\leq {}^1/_2\lambda_{\max}(M)\| e_s \|^2$$

in which M is a positive-definite matrix, accordingly, $\lambda_{\min}(M)>0$. The total time derivative of the scalar function V is given by

$$\begin{aligned}\dot{V}&=e_s^T M\dot{e}_s+{}^1/_2 e_s^T\dot{M}e_s\\&=e_s^T\{-Ce_s-(T_u+T_s)-Ke_s\\&\quad-\frac{\Phi_s^2 e_s}{\| e_s \|\Phi_s+\eta}\}+{}^1/_2 e_s^T\dot{M}e_s\end{aligned} \tag{12}$$

By using [P3], we obtain

$$\dot{V}\leq\| e_s \|\Phi_s-e_s^T Ke_s-\frac{\Phi_s^2\| e_s \|^2}{\| e_s \|\Phi_s+\eta} \tag{13}$$

from which

$$\begin{aligned}\dot{V}&\leq-e_s^T Ke_s-\frac{\eta\Phi_s\| e_s \|}{\| e_s \|\Phi_s+\eta}\\&\leq-K\| e_s \|^2+\eta\end{aligned} \tag{14}$$

Hence, we have the following differential inequality

$$\dot{V}\leq\nu V+\eta \tag{15}$$

with $\nu=2K/\lambda_{\max}(M)\geq 0$. It can easily be verified that the solution of (15), with $\eta\neq 0(\sigma>0,\ \kappa>0)$, can be written as

$$V\begin{cases}\leq\exp(-\nu t)[V_0-\sigma/(\nu-\kappa)]\\ +\exp(-\kappa t)[\sigma/(\nu-\kappa)],\ \nu\neq\kappa\\ \leq\exp(-\nu t)\ V_0+\sigma t\exp(-\nu t),\ \nu=\kappa\end{cases} \tag{16}$$

where $V_0=V_{t=0}(0,\ e_s(0))$ and ${}^1/_2\lambda_{\min}(M)\ \| e_s(0) \|^2\leq V_0\leq{}^1/_2\lambda_{\max}(M)\| e_s(0) \|^2$. From (16), it is shown that

$$\| e_s \|\begin{cases}\leq\sqrt{2/\lambda_{\min}(M)}\{\exp(-\nu t)[{}^1/_2\lambda_{\max}(M)\| e_s(0) \|^2\\ -\frac{\sigma}{\nu-\kappa}]+\exp(-kt)\frac{\sigma}{\nu-\kappa}\}^{1/2},\ \nu\neq\kappa\\ \leq\sqrt{2/\lambda_{\min}(M)}\{\exp(-\nu t)[{}^1/_2\lambda_{\max}(M)\| e_s(0) \|^2]+\sigma t\\ \exp(-\nu t)\}^{1/2},\ \nu=\kappa\end{cases} \tag{17}$$

Moreover, by Lemma 1, the uniform boundedness of e_s also guarantees those of e_p and $\dot{e}_v$. Thus from (16) and (17), we can conclude that the solutions (e_p, $\dot{e}_v$, e_s) are globally exponentially stable for any bounded initial values. If $\eta \neq 0$ ($\sigma > 0$, $\kappa = 0$), i.e., a saturation-type controller, then

$$V \leq \exp(-\nu t)[V_0 - \sigma/\nu] + \sigma/\nu \qquad (18)$$

Accordingly, the uniform ultimate boundedness (or UUB)(Chen, 1990 ; You, 1994c) results of system responses are achieved with respect to V_f. In other words, for $V_f = \sigma/\nu \geq 0$, V is nonincreasing, that is, $\dot{V} < 0$ for all $(t, e_s) \in R^+ \times R^n$ such that $V > V_f$ (or $V \subset \Lambda^c$). And its ultimate bound is given by $0 \leq \lim_{t \to \infty} V = \inf_t V = V_f \leq V_0$. Furthermore, the norm of joint tracking errors converges to the following ball, which is measure of the size of uniform ultimate boundedness :

$$\Omega(e_s) = \{e_s \in R^n : \| e_s \| \leq \sqrt{2\sigma/[\lambda_{\min}(M)\nu]}\}$$

from which one can show that $\| e_p \|$ and $\| \dot{e}_v \|$ are also ultimately bounded by

$$\Omega(e_p) = \{e_p \in R^n : \| e_p \| \leq (1/\varepsilon)\sqrt{2\sigma/[\lambda_{\min}(M)\nu]}\}$$
$$\Omega(\dot{e}_v) = \{\dot{e}_v \in R^n : \| \dot{e}_v \| \leq 2\sqrt{2\sigma/[\lambda_{\min}(M)\nu]}\}$$

This concludes the proof of the theorem. As a results, it is shown that all signals (or the tracking errors) in the closed-loop system are at least uniformly ultimately bounded.

Remarks :

1) For the specific value $\eta = 0$ (or $\sigma = 0$) in (15), i.e., purely discontinuous VS control law, it can be easily seen that the closed-loop system is also exponentially stable. That is, since V is nonincreasing function of t, which is upper bounded by V_0 and bounded from below on zero, i.e., $e_s \in L_\infty$, we have

$$0 \leq K \int_0^\infty \| e_s(\tau) \| \, d\tau \leq V_0 - \lim_{t \to \infty} V < \infty$$

which implies that $e_s \in L_2$. Now, we can obtain $\dot{e} \in L_\infty$ from Eq. (8), thus, $e_s \in L_2 \cap L_\infty$ and $\dot{e}_s \in L_\infty$. By Barbalat's lemma,(You, 1994c : Sastry, 1989) we can draw a conclusion that $e_s \to 0$, equivalently, $e_p \to 0$ and $\dot{e}_v \to 0$ as $t \to \infty$

2) Therefore, we have achieved stronger stability results in both $\eta = 0$(or $\sigma = 0$) and $\eta \neq 0$ ($\sigma > 0$, $\kappa > 0$), i.e., asymptotic stability, than those achieved in $\eta \neq 0$ ($\sigma \neq 0$, $\kappa = 0$).

4. Simulation Example

To illustrate the tracking capability of the control algorithms, we present a computer simulation example on a two-link mechanical manipulator (see Fig. 2) whose dynamic model can be expressed as(Ortega, 1989 ; You, 1994c)

$$\begin{aligned} T_1 = {} & m_2 l_2^2(\ddot{q}_1 + \ddot{q}_2) + m_2 l_1 l_2 c_2 (2\ddot{q}_1 + \ddot{q}_2) \\ & + (m_1 + m_2) l_1^2 \ddot{q}_1 - m_2 l_1 l_2 s_2 \dot{q}_2^2 \\ & - 2 m_2 l_1 l_2 s_2 \dot{q}_1 \dot{q}_2 + m_2 l_2 g c_{12} \\ & + (m_1 + m_2) l_1 g c_1 + T_{u1} \\ T_2 = {} & m_2 l_1 l_2 c_2 \ddot{q}_1 + m_2 l_2^2 (\ddot{q}_1 + \ddot{q}_2) \\ & + m_2 l_1 l_2 s_2 \dot{q}_1^2 + m_2 l_2 g c_{12} + T_{u2} \end{aligned}$$

where $s_i = \sin(q_i)$, $c_{ij} = \cos(q_i + q_j)$, etc. ; m_2 denotes the mass of the second link plus unknown payload.

The robot parameter vector Θ is defined as $\Theta = [l_1 l_2 m_1 m_2]^T$, and the corresponding actual values of Θ are assumed to be $l_1 = l_2 = 1(m)$ and $m_1 = m_2 = 1(kg)$. For simplicity, the nominal values are chosen by $\Theta_0 = [1.0\ 1.0\ 1.5\ 1.25]^T$. The initial conditions for the actual joint trajectories are given by $q_1(0) = q_2(0) = 0(rad)$ and $\dot{q}_1(0) = \dot{q}_2(0) = 0(rad/s)$, and the desired trajectories are supposed to be $q_{d1}(t) = q_{d2}(t) = 0.1 * \cos(10t)$. Let the system uncertainties be assumed to be $T_{ui} = q_i + \dot{q}_i + q_i \dot{q}_i + q_i^2 + \dot{q}_i^2 + \cos(\omega_f t)$, with $i(= 1, 2)$, where $\omega_f = 2.0$ or $100(rad/\text{sec})$. The known uncertainty bounds are given by $c = [10\ 10\ 10]^T$, where the highest order of polynomial bound in Θ_s is selected to be $m = 2$, i.e., qua-

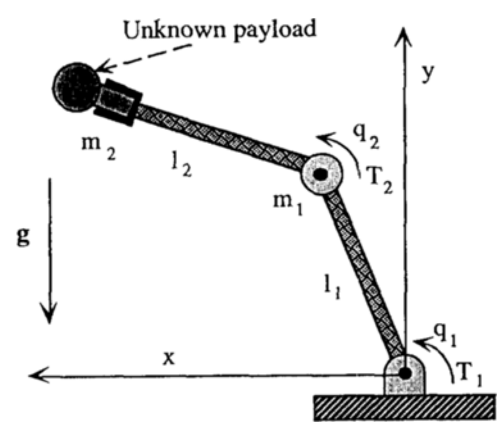

Fig. 2 Model of a two-link robot manipulator

dratically bounded uncertainties, however, more significant uncertainties may be assumed if necessary. The numerical values of other design variables are selected as $K=200$, $\varepsilon=2$, $\eta=0.7\,(\sigma=0.7,\ \kappa=0)$.

The simulation results are given in Figs. 3~9. Provided that the other design variables are unchanged, the chattering phenomena are observed with specific choices of $\eta=0.1$ (see Figs. 7~8) As a final remark, we also provide a simulation result to compare the performance of the proposed control laws with that of the PD controller ($\boldsymbol{T}=\boldsymbol{k}_p\boldsymbol{e}_p+\boldsymbol{k}_v\dot{\boldsymbol{e}}_v$), with control gains being the same (i.e., $\boldsymbol{k}_p=k_p\boldsymbol{E}_2$ and $\boldsymbol{k}_v=k_v\boldsymbol{E}_2$, with $k_p=400$ and $k_v=200$). It appears that the closed-loop system with the PD control is unstable (see Fig. 9). The results indicate that the control law gives better tracking properties than the PD control and that all signals of the corresponding closed-loop system are guaranteed to be

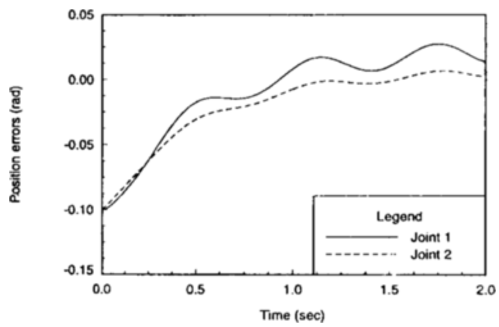

Fig. 5 Sliding velocity tracking errors ($\omega_f=100$, $\eta=0.7$)

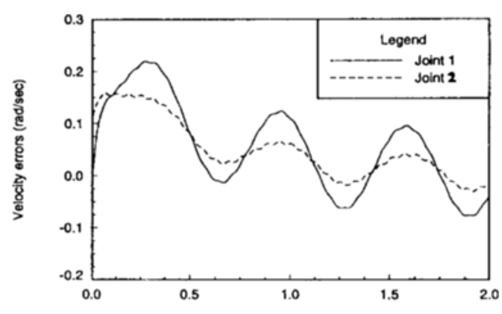

Fig. 4 Jointvelocitytrackingerrors ($\omega_f=100, \eta=0.7$)

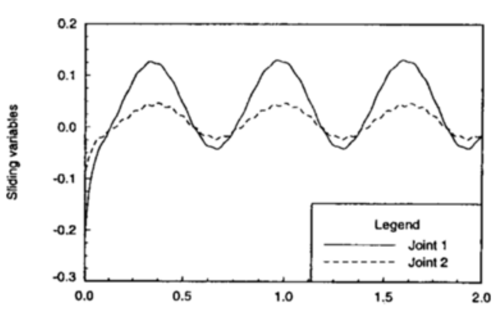

Fig. 3 Joint position tracking errors ($\omega_f=100$, $\eta=0.7$)

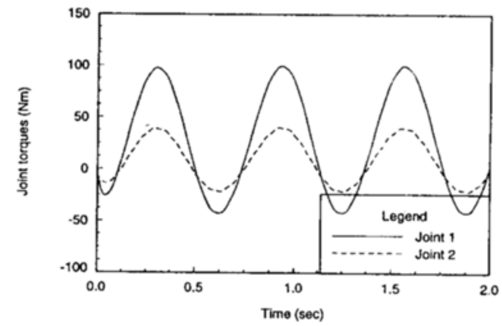

Fig. 6 Joint torgues ($\omega_f=100$, $\eta=0.7$)

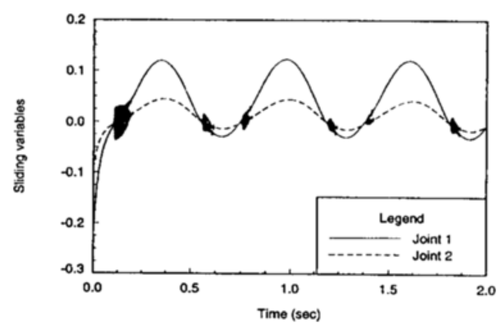

Fig. 7 Sliding variable tracking errors ($\omega_f=2.0$, $\eta=0.1$)

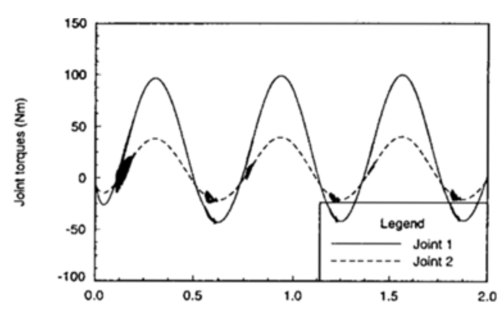

Fig. 8 Joint torques ($\omega_f=2.0$, $\eta=0.1$)

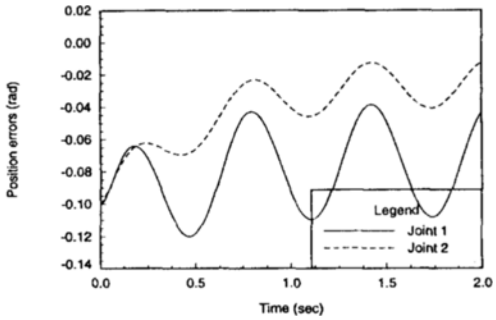

Fig. 9 Joint position tracking errors with PD controller

ultimately bounded under a given class of the uncertainties.

5. Conclusions

Based on a deterministic approach, we have addressed the robust trajectory tracking controllers for an uncertain robot system. It is shown that the control scheme consists of two major parts, that is, fully model-based feedforward plus PD compensation and robust nonlinear controller. The torque computations in the model-based feedforward part can be performed off-line since the desired trajectories (q_d, $\dot{q}_d$, $\ddot{q}_d$) and the nominal values of system parameters (Θ_0) are known in advance (or decentralized scheme), while many other methods rely heavily on the on-line computations to estimate the unknown system parameters (or centralized adaptive control). Both theoretical and simulation analysis are performed to verify the effectiveness of the suggested control algorithms. The results of this study can be summarized by pointing out that: the joint accelerations are not required in the control law; no exact knowledge as well as parametric values of the dynamic model are needed; the control torques in the model-based precompensation can be calculated off-line with less computational effort (this is particularly promising for real-time applications); the robust control part is intended to postcompensate for the effect of any higher-order uncertainties in the system; by Lyapunov stability method, we can conclude with certainty that the proposed control laws can guarantee at least the UUB results of all signals (or tracking errors) under the significant uncertainties.

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IEEE Int. SMC, Vol. 2, pp. 1279~1284.

Appendix

Proof of lemma 2

Noting from (9) that the model-mismatch uncertainty becomes

$$\begin{aligned}\|\boldsymbol{T}_s\| \le\ & \|\boldsymbol{M}-\boldsymbol{M}_0\|\|\ddot{\boldsymbol{e}}_r\| \\ & +\|\boldsymbol{C}-\boldsymbol{C}_0\|\|\dot{\boldsymbol{e}}_r\|+\|\boldsymbol{G}-\boldsymbol{G}_0\|\end{aligned}$$

Using [A2] and [P6], we can obtain the following inequality

$$\begin{aligned}\|\boldsymbol{T}_s\| &\le \rho_{11}\|\ddot{\boldsymbol{e}}_r\|+(\rho_{12}\|\dot{\boldsymbol{q}}\|+\rho_{13}\|\dot{\boldsymbol{q}}_d\|) \\ &\qquad \|\dot{\boldsymbol{e}}_r\|+\rho_{14} \\ &\le \rho_{11}(\|\dot{\boldsymbol{q}}_d\|+\mu\|\dot{\boldsymbol{e}}_v\|)+[\rho_{12}(\|\boldsymbol{q}_d\| \\ &\quad +\|\dot{\boldsymbol{e}}_v\|)+\rho_{13}\|\boldsymbol{q}_d\|](\|\boldsymbol{q}_d\| \\ &\quad +\varepsilon\|\boldsymbol{e}_p\|)+\rho_{14} \\ &\le [\rho_{11}d_3+(\rho_{12}+\rho_{13})d_2^2+\rho_{14}]+(\rho_{12}+\rho_{13}) \\ &\quad \varepsilon d_2\|\boldsymbol{e}_p\|+(\rho_{11}\varepsilon+\rho_{12}d_2)\|\boldsymbol{e}_v\| \\ &\quad +\varepsilon\rho_{12}\|\boldsymbol{e}_p\|\|\dot{\boldsymbol{e}}_v\|\end{aligned}$$

where $\|\dot{\boldsymbol{e}}_r\| \le \|\dot{\boldsymbol{q}}_d\|+\varepsilon\|\boldsymbol{e}_p\|$ and $\|\dot{\boldsymbol{q}}\| \le \|\dot{\boldsymbol{q}}_d\|+\|\dot{\boldsymbol{e}}_v\|$. In this way, by noting that $\|\boldsymbol{e}_p\| \le \|\bar{\boldsymbol{z}}\|$ and $\|\dot{\boldsymbol{e}}_v\| \le \|\bar{\boldsymbol{z}}\|$, the compensaion error can be finally estimated as

$$\|\boldsymbol{T}_s\| \le a_0+a_1\|\bar{\boldsymbol{z}}\|+a_2\|\bar{\boldsymbol{z}}\|^2$$

where

$$a_0=\rho_{11}d_3+(\rho_{12}+\rho_{13})d_2^2+\rho_{14}$$
$$a_1=(\rho_{12}+\rho_{13})\varepsilon d_2+\rho_{11}\varepsilon+\rho_{12}d_2 \text{ and}$$
$$a_3=\varepsilon\rho_{12}$$

The proof is completed.